GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

June 1986

Alberta EDUCATION

MATHEMATICS 30 MULTIPLE CHOICE KEY

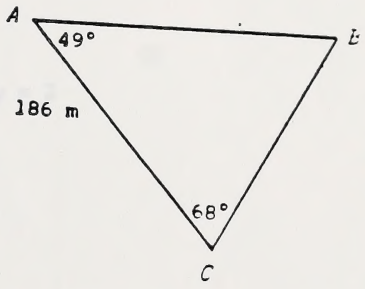
1.	C	27.	C
2.	B	28.	A
3.	D	29.	B
4.	B	30.	B
5.	A	31.	D
6.	B	32.	B
7.	C	33.	A
8.	C	34.	D
9.	A	35.	A
10.	B	36.	A
11.	D	37.	C
12.	C	38.	D
13.	C	39.	A
14.	C	40.	B
15.	D	41.	D
16.	D	42.	A
17.	D	43.	A
18.	A	44.	D
19.	D	45.	A
20.	D	46.	A
21.	D	47.	C
22.	C	48.	C
23.	A	49.	B
24.	B	50.	D
25.	A	51.	A
26.	B	52.	A

WPD#03490

SAMPLE ANSWER TO THE WRITTEN-RESPONSE SECTION

Note: The responses that follow represent ONE approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

(3 marks) 1. Trees A and B are separated by a body of water. To find the distance between them, a line AC, 186 m in length, is measured. Angles BAC and ACB are found to be 49° and 68° respectively. What is the distance from A to B? Give your answer to the nearest metre.



$\angle B = 63°$

$$\frac{AB}{\sin 68°} = \frac{186}{\sin 63°}$$

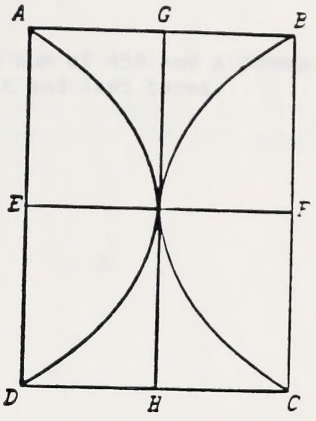
$$AB = \frac{186 \sin 68^\circ}{\sin 63^\circ}$$

$$= \frac{186(0.927)}{0.891}$$

$$= 193.5$$

$AB = 194$ m

(3 marks) 2. The sketch to the right shows a rectangle enclosing two parabolas, symmetrical with respect to line GH. If E and F are the foci and the equation of one parabola is $y^2 = 4x$, find the dimensions of the rectangle $ABCD$.



$4p = 4$

$p = 1$

$EF = AB = 2$

For $y^2 = 4x$ if $x = 1$ then $y = 2$

$AD = 2y = 4$

length = 4 units
width = 2 units

WPD#03490

(2 marks) 3. An arithmetic series of 9 terms has a sum of 459 and a common difference of 10. Determine the first and last terms.

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

$$459 = \frac{9}{2}\{2a + 8(10)\}$$

$$918 = 18a + 720$$

$$198 = 18a$$

$$11 = a$$

$$t_9 = 11 + 8(10)$$

$$= 91$$

$t_1 = 11,\ t_9 = 91$

(3 marks) 4. Baskets of grapes chosen at random contain a mean of 1000 grapes per basket with a standard deviation of 50. If the number of grapes per basket is distributed normally, find the probability of a basket containing between 900 and 1075 grapes.

$$z \text{ for } 900 = \frac{900 - 1000}{50} = -2$$

$$z \text{ for } 1075 = \frac{1075 - 1000}{50} = 1.5$$

Area for $-2\sigma = 0.4772$

Area for $+1.5\sigma = 0.4332$

Total area = 0.9104

probability is 0.9104

(2 marks) 5. Solve for x: $3^{\log_2(x)} = 9^{\log_4(16)}$

$$3^{\log_2(x)} = 9^2$$

$$3^{\log_2(x)} = 3^4$$

$$\log_2(x) = 4$$

$$2^4 = x$$

$x = 16$

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Five written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example	**Answer Sheet**
This examination is for the subject area of	A B C D
A. Chemistry **B.** Biology **C.** Physics **D.** Mathematics	① ② ③ ●

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JUNE 1986

PART A

INSTRUCTIONS

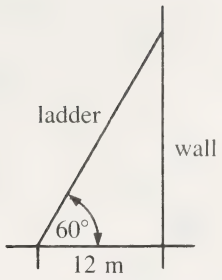
There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.

1. A ladder leans against a wall as shown in the diagram to the right. The length of the ladder is

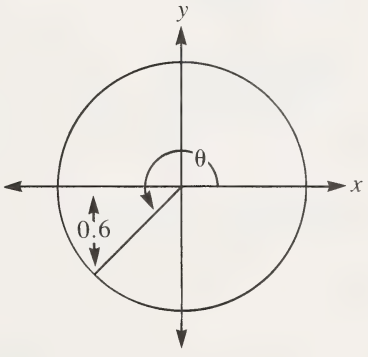


A. $8\sqrt{3}$ m
B. $12\sqrt{3}$ m
C. 24 m
D. $24\sqrt{3}$ m

2. The diameter of the unit circle is

A. 1
B. 2
C. π
D. 2π

3. In the unit circle to the right, the value of $\sin\theta$ is



A. -1
B. -0.8
C. -0.75
D. -0.6

4. In the domain $0 \leq \theta < 2\pi$, the set of solutions for the equation $\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}$ is

A. $0, \pi$

B. $\frac{\pi}{2}, \frac{3\pi}{2}$

C. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

D. $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

5. If $\sin\theta = -0.8$ and $\cos\theta < 0$, then the values of $\cos\theta$ and $\tan\theta$ are

A. $-0.6,\ 1.\overline{3}$

B. $0.6,\ -1.\overline{3}$

C. $-0.6,\ 0.75$

D. $0.6,\ 0.75$

6. If $0 < \theta < \frac{\pi}{2}$, then $\cot\theta$ expressed in terms of $\cos\theta$ is

A. $\dfrac{\cos\theta}{\sqrt{\cos^2\theta - 1}}$

B. $\dfrac{\cos\theta}{\sqrt{1 - \cos^2\theta}}$

C. $\dfrac{\sqrt{1 - \cos^2\theta}}{\cos\theta}$

D. $\dfrac{\sqrt{\cos^2\theta - 1}}{\cos\theta}$

7. The exact value of $\cos\left(\frac{\pi}{12}\right)$ is

A. $\dfrac{\sqrt{3}}{4}$

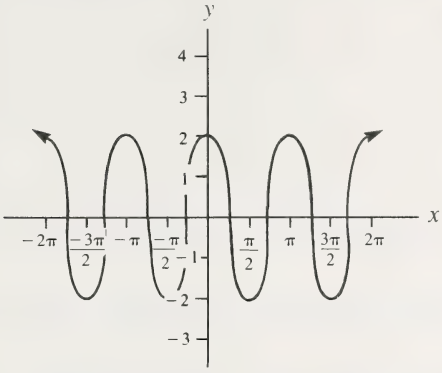
B. $\dfrac{\sqrt{2} - \sqrt{6}}{4}$

C. $\dfrac{\sqrt{2} + \sqrt{6}}{4}$

D. $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

8. Which sketch represents the graph of $y = \frac{1}{2}\cos x + 1$?

A.



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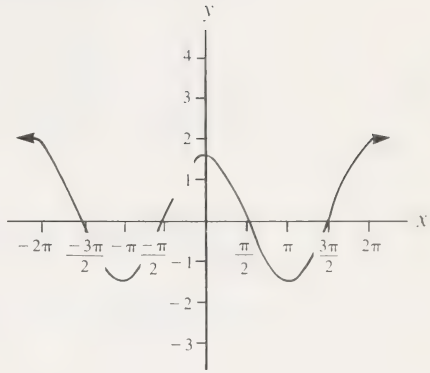

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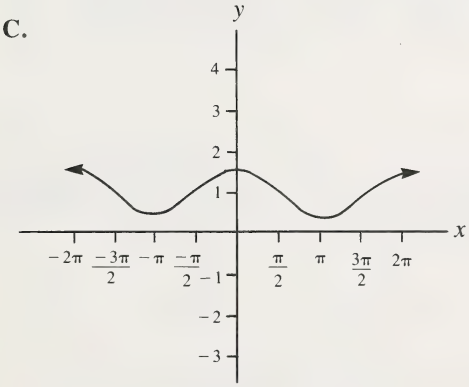

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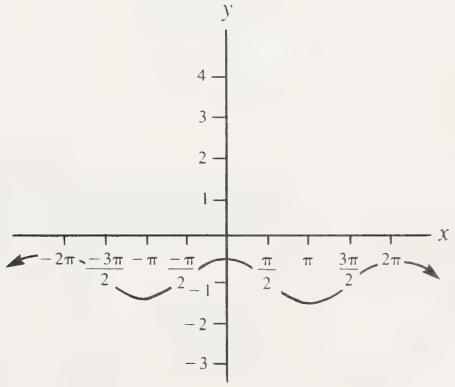


9. Expressed in radians, a measure of 235° equals

A. $\frac{47\pi}{36}$

B. $\frac{47\pi}{72}$

C. $\frac{36\pi}{47}$

D. $\frac{72\pi}{47}$

10. If $180° < \theta < 270°$, which statement gives the correct relationship between the values of $\sin \theta$ and $\tan \theta$?

A. $\sin \theta > \tan \theta$
B. $\sin \theta < \tan \theta$
C. $\sin \theta = \tan \theta$
D. $\sin \theta < -\tan \theta$

11. In ΔHAT, $\angle H = 130°$, $\angle A = 30°$, and side $a = 10$ cm. The measure of side h may be found by calculating

A. $5 \sin 130°$

B. $10 \sin 20°$

C. $10 \sin 130°$

D. $20 \sin 130°$

12. Two planes leave the same airport at the same time on two different runways and fly on courses that diverge by 40°. Their average speeds are 300 km/h and 600 km/h. To the nearest kilometre, how far apart are the planes after two hours?

A. 611 km
B. 772 km
C. 835 km
D. 920 km

13. A regular pentagon is inscribed in a circle of radius 4 cm. To the nearest tenth, the area of the pentagon in cm^2 is

A. 12.4 cm^2
B. 24.7 cm^2
C. 38.0 cm^2
D. 76.0 cm^2

14. The centre of the circle defined by $x^2 + y^2 + 4x - 6 = 0$ is

A. $(2, -3)$
B. $(2, 0)$
C. $(-2, 0)$
D. $(-2, 3)$

15. The equation of a circle with centre at $(3, -2)$ and tangent to the line given by $y - 3 = 0$ is

A. $(x + 3)^2 + (y - 2)^2 = 1$

B. $(x - 3)^2 + (y + 2)^2 = 5$

C. $(x + 3)^2 + (y - 2)^2 = 16$

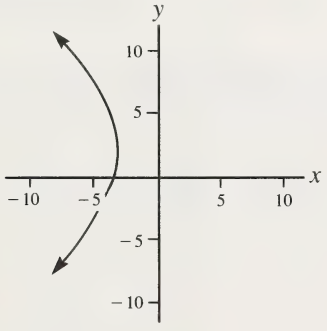
D. $(x - 3)^2 + (y + 2)^2 = 25$

16. The locus of all points equidistant from a fixed point P and a fixed line l is a

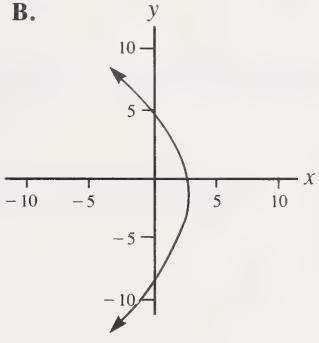
A. circle with centre at P and a tangent line l
B. hyperbola with focus at P and an asymptote l
C. parabola with vertex at P and a directrix l
D. parabola with focus at P and a directrix l

17. The sketch of the graph of $(x - 3)^2 = -20(y + 2)$ is

A.



B.



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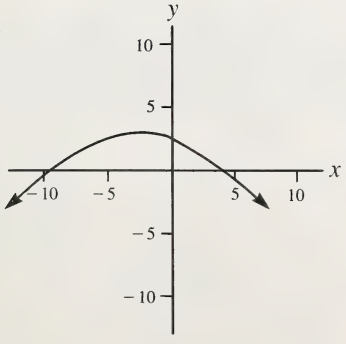

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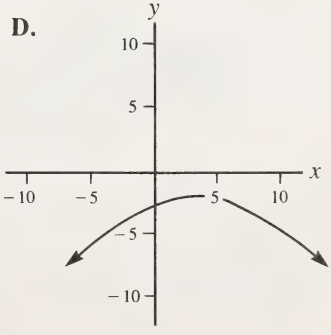


18. The parabola whose vertex is at the origin and whose focus is at the point $(-3, 0)$ is defined by the equation

A. $y^2 = -12x$

B. $y = -12x^2$

C. $y = 4(x + 3)^2$

D. $y^2 = 4(x + 3)$

19. Given that $a > b$, the equation of an ellipse with centre at $(0, 0)$ and minor axis along the x-axis is

A. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

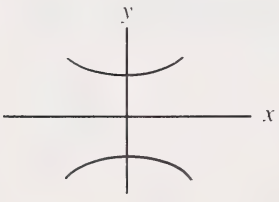
B. $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$

C. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

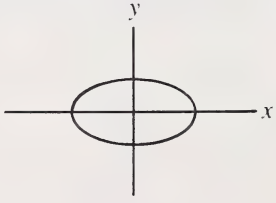
D. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

20. If $p > q$, $p,q \in N$, and $p^2x^2 = q^2(p^2 - y^2)$, then a sketch of the conic is

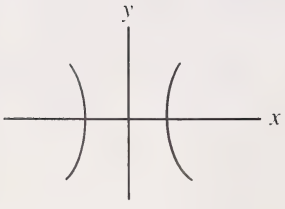
A.



B.



C.



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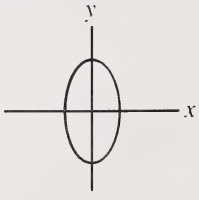


21. If the equation of an ellipse is $\frac{x^2}{64} + \frac{y^2}{100} = 1$, the length of the major axis is

A. 8
B. 10
C. 16
D. 20

22. The asymptotes of the hyperbola $\frac{x^2}{25} - \frac{y^2}{36} = 1$ are

A. $y = \pm \frac{36}{25} x$

B. $y = \pm \frac{25}{36} x$

C. $y = \pm \frac{6}{5} x$

D. $y = \pm \frac{5}{6} x$

23. If the hyperbola with centre at the origin and transverse axis along the x-axis passes through $(2, 0)$ and $(-3, 2)$, then its equation is

A. $4x^2 - 5y^2 = 16$

B. $4y^2 - 5x^2 = 16$

C. $x^2 - y^2 = 4$

D. $y^2 - x^2 = 4$

24. The path of a meteor is hyperbolic and may be represented graphically by a point moving so that the difference of its distances from $(0, 40)$ and $(0, -40)$ is always 60 units. If the meteor strikes an object located at $(5, y)$, the value of y is closest to

A. 31.5
B. 30.5
C. 29.5
D. 28.5

25. Which of the following is a finite sequence?

A. $\sqrt{3}, 3, 3\sqrt{3}, \ldots, (\sqrt{3})^n, \ldots, 27\sqrt{3}$

B. $16 + 14 + 12 + \ldots + (18 - 2n)$

C. $-5, 0, 5, \ldots, (5n - 10), \ldots$

D. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{3^n}, \ldots$

26. If the sum of 7 terms of an arithmetic series is 49 and the first term is -2, then the seventh term is

A. 18
B. 16
C. 12
D. 10

27. If 200 dollars are invested at 6 per cent per annum, compounded semi-annually for 10 years, then the compound amount will be

A. $200(1.03)^{10}$

B. $\frac{200}{(1.03)^{10}}$

C. $200(1.03)^{20}$

D. $\frac{200}{(1.03)^{20}}$

28. Using sigma notation, the series $4 + 9 + 16 + 25$ is

A. $\sum_{j=2}^{5} j^2$

B. $\sum_{j=2}^{5} 2^j$

C. $\sum_{j=1}^{4} (j^2 + 3)$

D. $\sum_{j=1}^{4} (3j + 1)$

29. As n approaches infinity, the limit of $\frac{1}{1 + \sqrt{n}}$ is

A. -1
B. 0
C. 1
D. undefined

30. The following: $\frac{1}{2}, -3, 18, -108, \ldots, \frac{(-6)^{n-1}}{2}, \ldots,$ is a

A. convergent sequence
B. divergent sequence
C. convergent series
D. divergent series

31. The $\lim_{n \to \infty} \left(\frac{5n}{n - 1} - \frac{n^2}{1 - n^2} \right)$ equals

A. -1
B. 1
C. 4
D. 6

32. The sum of $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \ldots + \frac{2^n}{(-3)^{n-1}} + \ldots$ is

A. 6

B. $1\frac{1}{5}$

C. $3\frac{1}{3}$

D. undefined

33. If the sum of an infinite geometric series is k times its first term, then r is

A. $1 - \frac{1}{k}$

B. $1 + \frac{1}{k}$

C. $k - 1$

D. $k + 1$

34. During the first swing of a pendulum the end point of the pendulum travels 100 cm in one direction. On each return swing the pendulum travels $\frac{49}{50}$ as far as in the previous swing. The distance the pendulum travels before coming to rest is

A. 10 000 cm
B. 9 900 cm
C. 9 800 cm
D. 5 000 cm

35. Three groups of scores have means of 7, 11, and 20 respectively. There are four times as many scores in the first group as there are in the second, and half as many scores in the third as there are in the second. The mean of the distribution of all the scores is

A. 8.9
B. 11.3
C. 12.7
D. 16.3

36. The mean of an examination was 18 and the standard deviation was 2. A student's z-score was -1.5. To the nearest integer, the examination mark was

A. 15
B. 17
C. 19
D. 21

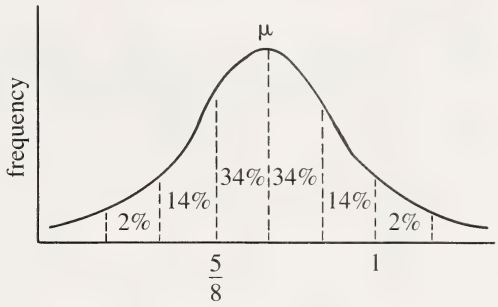
37. For the given normal distribution at the right, the value two standard deviations below the mean is

A. $\frac{1}{8}$

B. $\frac{3}{8}$

C. $\frac{1}{2}$

D. $\frac{7}{8}$



38. A normal distribution of marks resulted when 122 students wrote a Mathematics 30 exam. The number of students who achieved a mark within one standard deviation from the mean is

A. 15
B. 42
C. 61
D. 83

39. In a standard normal distribution, the percentage of scores that are lower than three standard deviations below the mean is

A. 0.1%
B. 2.5%
C. 49.9%
D. 99.9%

40. Two dice are rolled simultaneously. The probability that the numbers shown have a product that is larger than 29 is

A. $\frac{1}{8}$

B. $\frac{1}{12}$

C. $\frac{1}{9}$

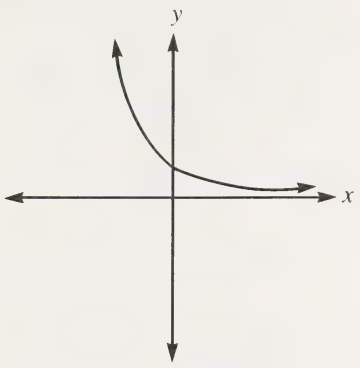
D. $\frac{11}{12}$

41. A certain normal distribution of numbers has a mean of 68 and a standard deviation of 11. The probability that a number selected at random from the distribution would be less than 115 and greater than 20 is

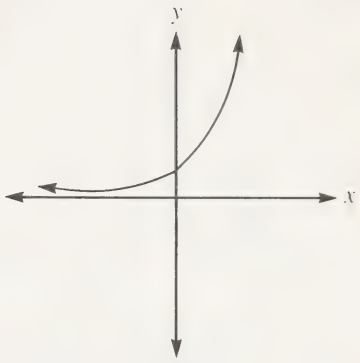
A. 0.09
B. 0.68
C. 0.95
D. 1.00

42. The sketch of the graph of $y = \left(\frac{1}{2}\right)^x$ is

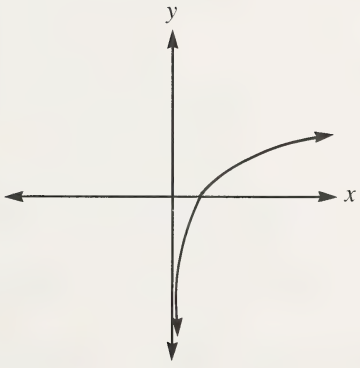
A.



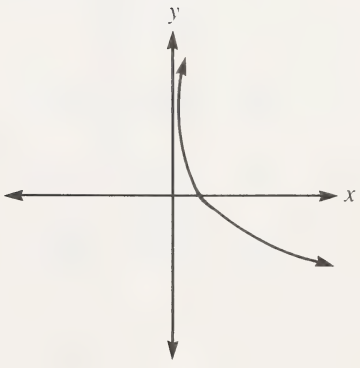
B.



C.



D.



43. If $\log_x(4) = -\frac{1}{2}$ then x equals

A. $\frac{1}{16}$

B. $\frac{1}{2}$

C. 2

D. 16

44. The value of $\log_2(0.125)$ is

A. $\frac{1}{8}$

B. -2

C. -8

D. -3

45. Expressed as a single logarithm, $5 \log_3(A) - \frac{1}{4}\log_3(B)$ is

A. $\log_3\left(\frac{A^5}{\sqrt[4]{B}}\right)$

B. $\log_3\left(\frac{5A}{\frac{1}{4}B}\right)$

C. $\log_3\left(5A - \frac{1}{4}B\right)$

D. $\log_3(A^5 - \sqrt[4]{B})$

46. If $x - 2$ is a factor of $6x^3 - 19x^2 + 11x + 6$, then the other two factors are

A. $(3x + 1)(2x - 3)$
B. $(3x - 1)(2x + 3)$
C. $(3x + 3)(2x - 1)$
D. $(3x - 3)(2x - 1)$

47. Which of the following is a polynomial of degree 4?

A. $\dfrac{2}{x^4 - 1}$

B. $2x^4 + 5 - \dfrac{1}{2}x^5$

C. $x^2 - \dfrac{1}{3}x + 3x^4 + 1$

D. $x^4 - 3x^{-3} + 2x^2 + 5x - 6$

48. If $x^4 + 5x^2 - 6$ is divided by $x - 1$, the number of terms in the quotient is

A. 2
B. 3
C. 4
D. 5

49. If the polynomial $P(x) = 15x^3 + 13x^2 - 9x + 1$, the value of $P\left(\dfrac{1}{3}\right)$ is

A. -6
B. 0
C. 2
D. 5

50. A polynomial whose zeros are $-2 + a$, $-2 - a$, and $\dfrac{3}{2}$ is

A. $(3x - 2)(x - 2 + a)(x - 2 - a)$
B. $(3x - 2)(x + 2 + a)(x + 2 - a)$
C. $(2x - 3)(x - 2 + a)(x - 2 - a)$
D. $(2x - 3)(x + 2 + a)(x + 2 - a)$

51. The graph of $P(x) = 4x^3 - 6x^2 - 16x - 6$ intersects the x-axis three times. If two of the intercepts are $x = -1$ and $x = 3$, then the third one is

A. $x = -\frac{1}{2}$

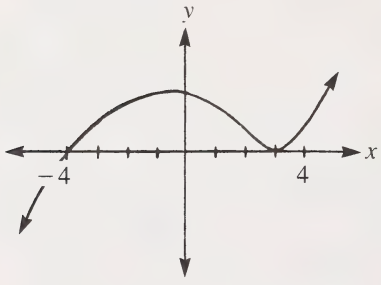
B. $x = 0$

C. $x = \frac{1}{2}$

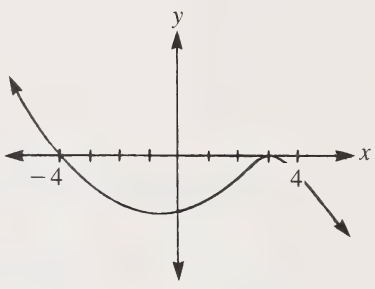
D. $x = 2$

52. The sketch which represents $P(x) = (x - 3)^2 (x + 4)$ is

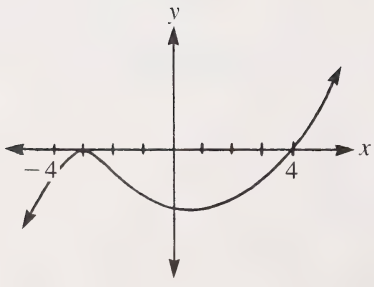
A.



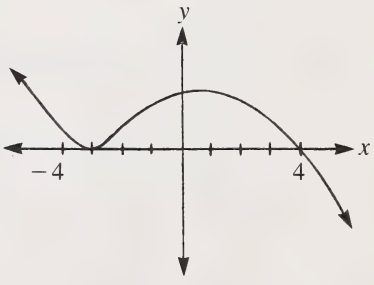
B.



C.



D.



YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas. Part marks will be given for partial solutions.

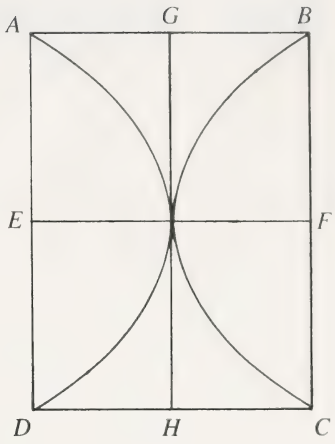
NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

TOTAL MARKS: 13

START PART B IMMEDIATELY

(3 marks) 1. Trees A and B are separated by a body of water. To find the distance between them, a line AC, 186 m in length, is measured. Angles BAC and ACB are found to be 49° and 68° respectively. What is the distance from A to B? Give your answer to the nearest metre.

(3 marks) 2. The sketch to the right shows a rectangle enclosing two parabolas that are symmetrical with respect to line GH. If E and F are the foci, and the equation of one parabola is $y^2 = 4x$, find the dimensions of the rectangle $ABCD$.



(2 marks) 3. An arithmetic series of 9 terms has a sum of 459 and a common difference of 10. Determine the first and last terms.

(3 marks) 4. Baskets of grapes chosen at random contain a mean of 1000 grapes per basket with a standard deviation of 50. If the number of grapes per basket is distributed normally, find the probability of a basket containing between 900 and 1075 grapes.

(2 marks) 5. Solve for x: $3^{\log_2(x)} = 9^{\log_4(16)}$

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION

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M1 M2 M3 M4

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MATHEMATICS 30

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